Theoretical Analysis of Magnetic Resonance Nonreciprocal Circuits — Limitations of 3-dB Bandwidth and Available Range

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Abstract—The general formulas for both 4-port magnetic-resonator circulators and 2-port magnetic-resonator bandpass filters (BPF's) are given. The effects of the circuit structure and the physical properties of the resonator on the characteristics of the circuit are investigated.

The maximum dissipation loss of the resonator is just one half of the input power. Contrary to the results shown in previous reports, it is understood that for a given resonator, there exists an optimum value of the external Q of the circuit and also that a polycrystalline magnetic resonator may be useful for the nonreciprocal circuit, if the product of unloaded Q and saturation magnetization has a sufficiently large value.

The requirements for the insertion loss (IL), the reverse loss (RL), and the reflection at the center frequency are introduced, and then the necessary conditions of resonator loss and the limitations of 3-dB bandwidth and available range are clarified.

I. INTRODUCTION

SO FAR, a large number of studies concerned with magnetically tunable ferrimagnetic resonance filters have been reported. The 4-port YIG circulator in waveguide and its scattering matrix were reported by Skeie [1]. A 4-port nonreciprocal circuit in strip line developed by the authors is shown in Fig. 1. The circuit consists of two directional couplers and an X-form orthogonal coupling with a magnetic resonator placed in the center of the circularly polarized RF magnetic field. An external dc magnetic field H is applied perpendicular to the RF field. The circuit operates over a very wide frequency range as a magnetically tunable resonance-type circulator [2]. An example of the experimental results of this 4-port circulator is shown in Fig. 2(a), and the center frequency insertion losses (IL's) in the passband and in the stopband are defined in Fig. 2(b). These definitions will be used in the case of the 2-port bandpass filter (BPF), too.

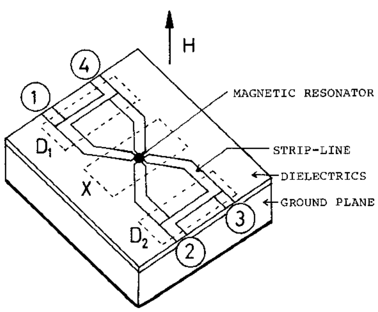

Fig. 1. Schematic 4-port nonreciprocal circuit. D_1, D_2: 3-dB directional coupler; X: X-form orthogonal coupling.

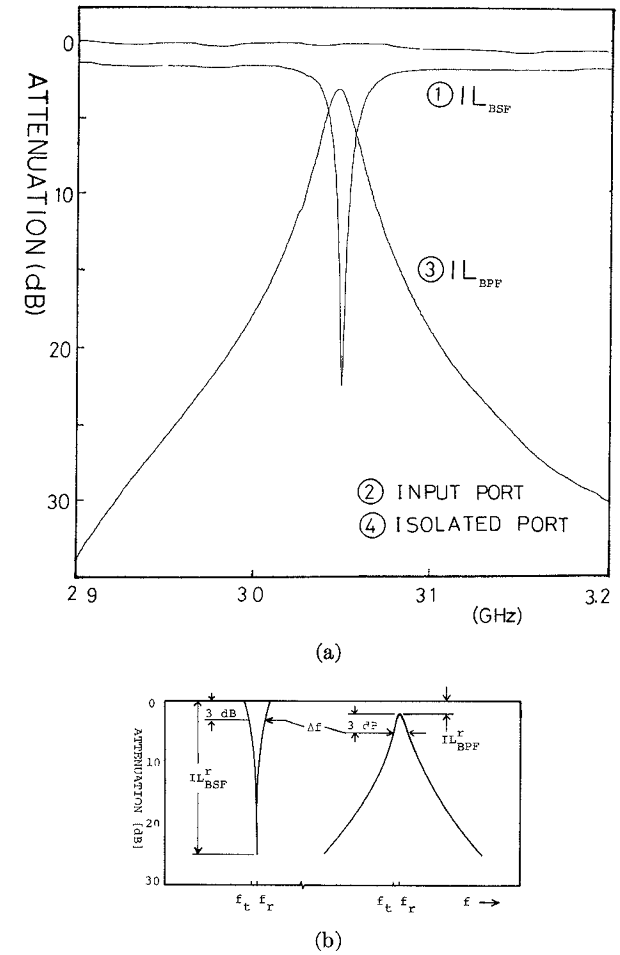

Fig. 2. (a) Output response of the 4-port circulator. A dc magnetic field is tuned to the YIG sphere resonance and fixed. Conditions: single YIG sphere, diameter: 1 mm, $4\pi M_s$ = 1750 G, Q_u = 1530 ($\alpha = 3.26 \times 10^{-4}$). (b) Definition of filter response parameters.

The fundamental type of 2-port filter is shown in Fig. 3, which consists of two concentric coupling loops arranged at right angles to each other, with an ellipsoidally shaped ferrite placed in the center of the loops, and an external dc magnetic field H applied along the z axis. A 2-port ferrimagnetic resonance filter circuit which made use of this coupling principle was first proposed and demonstrated by DeGrasse [3] and later analyzed by Carter [4]. Recently, interesting results of numerical analysis on this 2-port filter were reported by Carter [5] and by Bex [6]. The 2-port circuit was applied as a ferrite tuner

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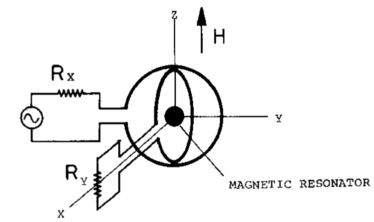


Fig. 3. Fundamental type of 2-port magnetic resonator BPF.

with polycrystalline CaV garnet at UHF band by Konishi and Utsumi [7], which was, at the same time, analyzed by them.

The orthogonal coupling of the 4-port circulator in Fig. 1 or that of the 2-port filter in Fig. 3 can be represented as shown in Fig. 4. That is, when a pair of ports 1 and 3 is regarded as an input port (where $R_x = Z_{01} + Z_{03}$) and a pair of ports 2 and 4 as an output port (where $R_y = Z_{02} + Z_{04}$), this 2-port circuit becomes equivalent to the 2-port BPF in Fig. 3. Therefore, the results of analyzing an X-form orthogonal coupling shown in Fig. 4 can be applied not only to the 4-port nonreciprocal circuit but also to the 2-port filter. There are, however, essential differences in the characteristics of the 4-port circuit and the 2-port one. The 4-port circuit is nonreciprocal in the sense of amplitude, that is, there is a small attenuation in one direction of propagation and a large attenuation in the reverse direction. On the other hand, the 2-port circuit is not nonreciprocal in that sense. But in the sense of phase shift, it is nonreciprocal, that is, there is only a phase shift between one direction of propagation and the other direction. Therefore, in the case of the 4-port nonreciprocal circuit, the limitations of available frequency range and 3-dB bandwidth are clarified, taking not only the IL but also the reverse loss (RL) into account. Thus there are some differences between the limitations of the 4-port circuit and those of the 2-port one which will be discussed in Section V, for instance, the upper limitations of available frequency range in (19) and (23).

In the case of the 4-port nonreciprocal circuit shown in Fig. 1, its complete analysis has not been reported yet. In the case of 2-port magnetic-resonator BPF's, it has been pointed out, in previous reports (for instance, [8]), that the center frequency IL in the BPF can be reduced by higher unloaded Q of the magnetic resonator or by stronger coupling between the circuit and the resonator. Carter [5] analyzed the frequency responses (with some parameters) of the 2-port BPF in detail, under the condition that the RF losses in the ferrite can be neglected. Bex [6] and Konishi *et al.* [7] analyzed the same characteristics, taking into account the RF losses of the resonator. It seems, however, that the relations between filter characteristics and physical properties of the resonator have not been explained sufficiently.

The purpose of this paper is to give the theoretical formulas for the response of a 4-port and a 2-port nonreciprocal circuit and to investigate the effects of the circuit structure and the physical properties of the resonator[1] on the characteristics of the circuit, in particular, at the resonance frequency. For this purpose, we divide the IL in the passband into the net dissipation losses of the resonator and the remaining losses (for instance, reflections at the input port in the case of a 2-port filter). One of the results obtained from a complete analysis shows that a phenomenon of interest occurs when the external Q is equal to the unloaded Q of the resonator; in this case, the net dissipation losses of the resonator have a maximum, a value which is just one half of the input. The center frequency IL in the passband is, at the same time, equal to that in the stopband (4-port circuit) or to the reflection at the input port (2-port filter).

It is explicitly shown that there exists an optimum value of the external resonator Q, when the resonator parameters (for instance, loss term α, or saturation magnetization M_s) are given. In the theoretical analysis, the requirements for the characteristics of the IL or that of the reflection at the input port are introduced, and then the conditions of the resonator and circuit structure to satisfy these requirements are discussed. As a consequence, it has become clear that even a polycrystalline resonator could be satisfactorily used for most purposes. The desirable physical property of a resonator (for the nonreciprocal circuits mentioned here) is to have a large value of the product of unloaded Q and saturation magnetization.

The results also show the limitations of the 3-dB bandwidth and tunable frequency range. The basic principle of design for these nonreciprocal circuits can be obtained from these results of the complete analysis.

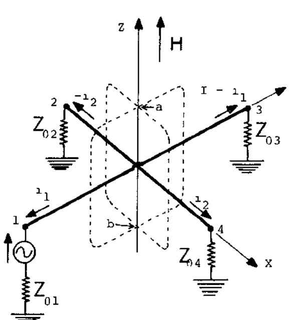


Fig. 4. Schematic X-form orthogonal coupling. Positions a and b show the center of the circularly polarized RF magnetic field. A magnetic resonator is placed in the position a (or b).

II. SCATTERING MATRIX OF AN X-FORM ORTHOGONAL COUPLING

In this section, the scattering matrix of an X-form orthogonal coupling is discussed and the results in the case of symmetric coupling lines will be summarized. In the following analysis, the magnetic resonator is assumed operating in the uniform precessional mode; the (110) mode. Therefore, higher order modes are not excited (refer

[1] Circuit structure: for instance, external Q of the resonator and self-inductance of coupling line L_c. Physical properties: Landau–Lifshitz phenomenological loss term α and saturation magnetization M_s.

to Section IV-1). Furthermore, magnetocrystalline anisotropy, ohmic losses in strip lines, and radiation losses are neglected. As shown in Fig. 4 ports 1–4 are terminated with an impedance Z_{01}, Z_{02}, Z_{03}, and Z_{04}, respectively, and a magnetic resonator is placed in the position a (or b). An external dc magnetic field H is applied along the z direction. When current I flows on the 1-3 line (x line which means that the current of the 1-3 line induces the x component of RF magnetic field), the reflected current i_1 due to the magnetic resonator and the coupled current i_2 flowing on the 2-4 line (y line which means that the current of the 2-4 line induces the y component of RF magnetic field) are related to one another in simultaneous equations as follows:

$$\left(1+\frac{Z_{xx}}{Z_{01}+Z_{03}}\right)i_1+\frac{Z_{xy}}{Z_{01}+Z_{03}}i_2=\frac{Z_{xx}}{Z_{01}+Z_{03}}I$$

$$\frac{Z_{yx}}{Z_{02}+Z_{04}}i_1+\left(1+\frac{Z_{yy}}{Z_{02}+Z_{04}}\right)i_2=\frac{Z_{yx}}{Z_{02}+Z_{04}}I \quad (1)$$

where

- Z_{xx} $j\omega L_{cx}+j\omega A_{xx}\chi_{xx}^e$;
- Z_{xy} $j\omega A_{xy}\chi_{xy}^e$;
- Z_{yx} $j\omega A_{yx}\chi_{yx}^e$;
- Z_{yy} $j\omega L_{cy}+j\omega A_{yy}\chi_{yy}^e$;
- ω radian frequency of input signal;
- L_{cx} self-inductance of the x line (1-3 line);
- L_{cy} self-inductance of the y line (2-4 line);
- χ_{xx}^e, χ_{xy}^e, χ_{yx}^e, χ_{yy}^e components of RF external tensor susceptibility of the magnetic sample;
- A_{xx}, A_{xy}, A_{yx}, A_{yy} inductance depending on the volume of the magnetic sample, the position of the magnetic sample, and the geometrical configuration of an orthogonal coupling (Appendix).

The scattering matrix of an orthogonal coupling can be obtained in a general form from the solution (i_1 and i_2) of simultaneous equations (1). In this study, we will analyze an X-form orthogonal coupling for the symmetric case where $A_{xx}=A_{xy}=A_{yx}=A_{yy}=A$ and $L_{cx}=L_{cy}=L_c$. Taking the symmetry of this circuit into account and by means of the scattering matrix, the input and output responses are directly obtained from (1):

$$\begin{pmatrix} b_1\\ b_2\\ b_3\\ b_4\end{pmatrix}=\begin{pmatrix}\Gamma & S_{12} & T & S_{21}\\ S_{21} & \Gamma & S_{12} & T\\ T & S_{21} & \Gamma & S_{12}\\ S_{12} & T & S_{21} & \Gamma\end{pmatrix}\begin{pmatrix} a_1\\ a_2\\ a_3\\ a_4\end{pmatrix} \quad (2a)$$

where a_i and $b_i (i=1,2,3,4)$ are an input wave and an output one at port "i," respectively, and

$$\Gamma=\frac{1}{\Delta(Z_{01}+Z_{03})}\left(Z_{xx}+\frac{Z_{xx}Z_{yy}-Z_{xy}Z_{yx}}{Z_{02}+Z_{04}}\right) \quad (2b)$$

$$T=\frac{1}{\Delta}\left(1+\frac{Z_{yy}}{Z_{02}+Z_{04}}\right) \quad (2c)$$

$$S_{12}=-S_{21}=\frac{Z_{yx}}{\Delta(Z_{02}+Z_{04})} \quad (2d)$$

$$\Delta=\left(1+\frac{Z_{xx}}{Z_{01}+Z_{03}}\right)\left(1+\frac{Z_{yy}}{Z_{02}+Z_{04}}\right)-\frac{Z_{xy}}{Z_{01}+Z_{03}}\cdot\frac{Z_{yx}}{Z_{02}+Z_{04}}.$$

For the commonly employed case where $Z_{01}=Z_{02}=Z_{03}=Z_{04}=Z_0$ and where the resonator is of the ellipsoidally shaped sample, the components of the scattering matrix become as follows:

$$\Gamma=\frac{1}{2}\left(\frac{Z_+}{1+Z_+}+\frac{Z_-}{1+Z_-}\right) \quad (3a)$$

$$T=\frac{1}{2}\left(\frac{1}{1+Z_+}+\frac{1}{1+Z_-}\right) \quad (3b)$$

$$S_{12}=-S_{21}=j\frac{1}{2}\left(\frac{1}{1+Z_+}-\frac{1}{1+Z_-}\right) \quad (3c)$$

where

$$Z_\pm=j\frac{1}{Z_{00}}\left(\omega L_c+\frac{x}{\mp1+\omega_t/\omega+j\alpha}\right)$$

Z_+ and Z_- are connected with a clockwise circularly polarized magnetic field and a counterclockwise one, respectively,

$$Z_{00}=\begin{cases}R_0, & \text{for 2-port filter circuit}\\ 2Z_0, & \text{for 4-port circulator circuit}\end{cases}$$

$x=A\omega_m$ (Ω), which is in this discussion defined as a "coupling impedance" related with a strength of coupling between the resonator and the coupling circuit, $\omega_m=-\gamma(M_s/\mu_0)$, γ is the gyromagnetic ratio, μ_0 is the intrinsic permeability of free space, $\omega_t=\omega_0-(1-3N_t)\omega_m$, $\omega_0=-\gamma H$, and N_t is the transverse component of the demagnetizing factor.

In order to simplify the analysis, the following equations are introduced here:

$$Q_f=\frac{Z_{00}}{x} \quad \text{(external } Q \text{ of magnetic resonator in the case of 4-port circuit[2])} \quad (4a)$$

[2] This is the single-line external Q due to impedances at both ports of the line in question. The external Q of the circuit would be half this value.

$$k = \frac{\omega L_c}{x} \quad (4b)$$

$$Q_e = Q_f\left[1 + \left(\frac{k}{Q_f}\right)^2\right] \quad \text{(external } Q \text{ of magnetic resonator in the case of 2-port circuit}^2\text{)} \quad (4c)$$

$$P_\pm = \left[Z_{00}\left(\frac{\omega_t}{\omega} \mp 1\right) - \alpha\omega L_c\right] + j\left[\omega L_c\left(\frac{\omega_t}{\omega} \mp 1\right) + x + \alpha Z_{00}\right]. \quad (4d)$$

III. 4-PORT CIRCULATOR

1) *General Formula:* We investigated previously a 4-port nonreciprocal circuit in the sense that there is a small attenuation in one direction of propagation, and a large attenuation in the reverse direction [2] in Fig. 1. In this paper, we will discuss in detail the properties of the circuit which have not been reported yet. Equation (2a) shows only the characteristics of X-form orthogonal coupling. In order to understand the characteristics of a 4-port circuit in Fig. 1, we take account of two directional couplers connected with X-form orthogonal coupling. After consideration to (2a) and the scattering matrix of the 3-dB directional coupler, the behaviors of the 4-port nonreciprocal circuit can be summerized as follows.

a) When port 1 is an input port (a magnetic resonator sees a counter-clockwise circularly polarized RF field),

$$b_{11} = 0$$

$$b_{21} = -jT - S_{21} \quad \text{[reverse direction of the bandstop filter (BSF)]}$$

$$b_{31} = 0$$

$$b_{41} = -j\Gamma + S_{21} \quad \text{(reverse direction of the BPF)}$$

where b_{ij} is an output response at port "i" in the case where a unit input is applied at port "j." As known from (3a)–(3c), $b_{21} + b_{41} = -j$.

b) When port 2 is an input port (a magnetic resonator sees a clockwise circularly polarized RF field),

$$b_{12} = -jT - S_{12} \quad \text{(BSF)}$$

$$b_{22} = 0$$

$$b_{32} = -j\Gamma + S_{12} \quad \text{(BPF)}$$

$$b_{42} = 0.$$

Also, $b_{12} + b_{32} = -j$.

c) From the symmetry of the circuit,

$$b_{13} = b_{31} = 0$$

$$b_{23} = b_{41} \quad \text{(reverse direction of the BPF)}$$

$$b_{33} = 0$$

$$b_{43} = b_{21} \quad \text{(reverse direction of the BSF)}$$

$$b_{14} = b_{32} \quad \text{(BPF)}$$

$$b_{24} = b_{42} = 0$$

$$b_{34} = b_{12} \quad \text{(BSF)}$$

$$b_{44} = 0.$$

Note that b_{ij} gives an "i, j" element of the whole scattering matrix of the 4-port nonreciprocal circuit in Fig. 1.

The IL in the passband $\mathrm{IL_{BPF}}$, the IL in the stopband $\mathrm{IL_{BSF}}$, and the net dissipation loss L of the magnetic resonator can be, respectively, expressed as follows:

$$\mathrm{IL_{BPF}} = \frac{1}{|b_{32}|^2} = \left|1 + \frac{1}{Z_+}\right|^2 = \frac{|P_+|^2}{[x + \omega L_c(\omega_t/\omega - 1)]^2 + (\alpha\omega L_c)^2} \quad (5a)$$

$$\mathrm{IL_{BSF}} = \frac{1}{|b_{12}|^2} = |1 + Z_+|^2 = \frac{|P_+|^2}{[2Z_0(\omega_t/\omega - 1)]^2 + (\alpha 2Z_0)^2} \quad (5b)$$

$$L = 1 - |b_{12}|^2 - |b_{32}|^2 = \frac{4\alpha x Z_0}{|P_+|^2} \quad (5c)$$

where an input port is assumed to be matched.

2) *Analysis at Resonance:* We will investigate the characteristics of the circuit, in particular, at the resonant frequency. Discussions from this point of view are very important to clarify the limitations of the magnetic-resonator circuits. From the conditions of the minimum IL in the passband[3] ($(d/d\omega)\mathrm{IL_{BPF}} = 0$) and $k_r^2(Q_uQ_f)^{-2} \ll 1$, the resonant frequency f_r can be determined by the following relation:

$$f_r = f_t\left(1 + \frac{k_r}{Q_uQ_f}\right)^{-1} \simeq f_t\left(1 - \frac{k_t}{Q_uQ_f}\right) \quad (6)$$

where k_i is the value of k at $\omega = \omega_i (i = r,t)$. Therefore, at resonance, equations (5a)–(5c) become as follows:

$$\mathrm{L_{BPF}}^r = \left|1 + \frac{1}{Z_+^r}\right|^2 \simeq (1 + \alpha Q_f)^2 \quad (7a)$$

$$\mathrm{IL_{BSF}}^r = |1 + Z_+^r|^2 \simeq \left(1 + \frac{1}{\alpha Q_f}\right)^2 \quad (7b)$$

$$L^r = \frac{4\alpha x Z_0}{|P_+^r|^2} \simeq \frac{2\alpha Q_f}{(1 + \alpha Q_f)^2} \quad (7c)$$

and the RL of the BPF $\mathrm{RL_{BPF}}^r$ and that of the BSF $\mathrm{RL_{BSF}}^r$ at the center frequency are, respectively, expressed as follows:

[3] From $(d/d\omega_t)$ $\mathrm{IL_{BPF}} = 0$, we get (6), too where the $\mathrm{IL_{BPF}}$ is a function of ω_t for a given ω_r.

$$\mathrm{RL_{BPF}}^r = \left|1 + \frac{1}{Z_-{}^r}\right|^2 \simeq 1 + \left(\frac{2Q_f}{2k_r + 1}\right)^2 \quad (8a)$$

$$\mathrm{RL_{BSF}}^r = |1 + Z_-{}^r|^2 \simeq 1 + \left(\frac{2k_r + 1}{2Q_f}\right)^2 \quad (8b)$$

where index r means resonance and the approximate expressions hold in the case where $k_r^2(Q_uQ_f)^{-1} \ll 1$. The characteristics of the nonreciprocal BPF and the net dissipation loss of the magnetic resonator are shown as a function of αQ_f in Fig. 5, where the product αk_r is a parameter. Also shown in Fig. 6 are the characteristics of the nonreciprocal BSF.

3) The dissipation loss of the resonator goes through the maximum 1/2 at a value of $Q_f = 1/\alpha$, after which it decreases and at last approaches a value of zero with increasing Q_f. In this 4-port nonreciprocal circuit, the

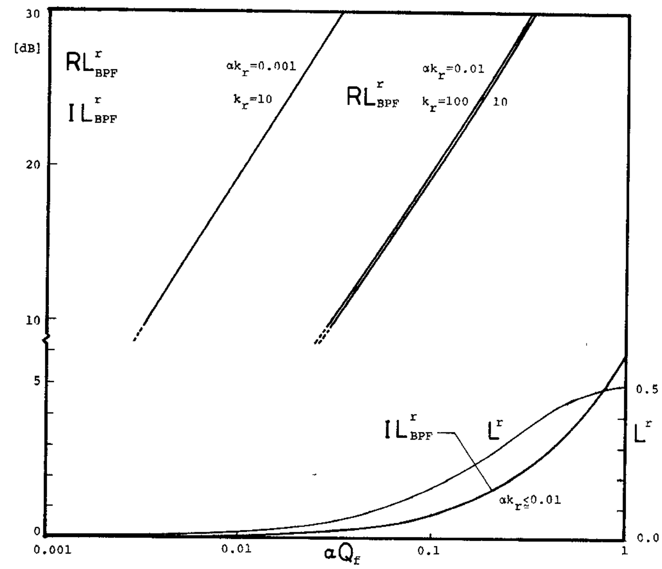


Fig. 5. IL and RL in the passband and dissipation loss of 4-port nonreciprocal circuit as a function of αQ_f at resonance.

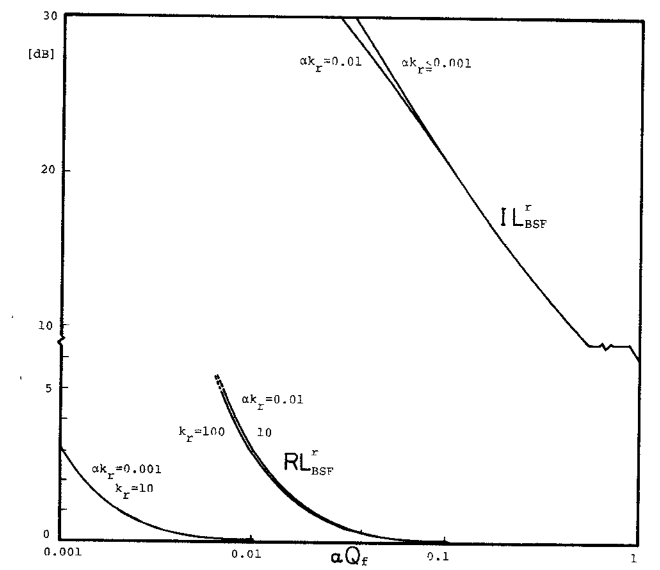


Fig. 6. IL and RL in the stopband of the 4-port nonreciprocal circuit as a function of αQ_f at resonance.

reflections at input port are equal to zero and the IL in the stopband decreases, with an increase of that in the passband and the net dissipation loss. In relatively low regions of αQ_f, even if an α of the resonator has a smaller value, the center frequency RL in the passband decreases and that in the stopband increases with increasing k_r. These effects are considered to degrade the characteristics of the 4-port circuit in higher regions of available frequency range. The center frequency IL in the passband increases with increasing α, while the center frequency IL in the stopband decreases in spite of increasing the dissipation loss. Thus there exists an optimum value of external Q, Q_f, to hold a small value of the $\mathrm{IL_{BPF}}^r$ and $\mathrm{RL_{BSF}}^r$ and, at the same time, a relatively large value of the $\mathrm{IL_{BSF}}^r$ and $\mathrm{RL_{BPF}}^r$.

4) As understood from Figs. 5 and 6, the circuit structure is fixed (i.e., A = constant and Z_0 = constant), the center frequency RL in the passband is not always increased, and that in the stopband is not always reduced, though the resonator with smaller α is used. Thus it is a key point whether the Q_f and k depending on the M_s increase or decrease. Consequently, for 4-port circulators of this type, the magnetic materials which have a large value of the product of Q_u and M_s are of much importance as magnetic resonators. Also, the circuit structure which reduces a value of the ratio L_c/A is required. In such a case as the ratio L_c/A can be reduced sufficiently, it is considered that good performances of the circuit may be obtained by using even a polycrystalline magnetic resonator, if the product $Q_u \cdot M_s$ is adequately chosen.

IV. 2-PORT BPF

1) *General Formula:* As mentioned in Section I the 2-port nonreciprocal BPF circuit (in the sense that there is a +90° phase shift in one direction of propagation, and a −90° phase shift in the other direction) can be obtained from an X-form orthogonal coupling circuit in Fig. 4. Let us consider a pair of ports 1 and 3 as an input port and a pair of ports 2 and 4 as an output port. For the case where the source resistance R_x and the load resistance R_y are equal to R_0 and the conditions mentioned in Section II are satisfied,[4] the analysis of this 2-port circuit[5] yields the following formula for the IL in the passband, i.e., inverse of $|b_{21}|^2$:

$$\mathrm{IL} = \left|\frac{P_+P_-}{2R_0x}\right|^2. \quad (9)$$

In particular when $L_c = 0$, the IL in the passband approaches a maximum value as $\omega \to \infty$ given by

$$\lim_{\omega\to\infty} \mathrm{IL} = \frac{1}{4}\left[Q_f + \frac{(1+\alpha Q_f)^2}{Q_f}\right]^2. \quad (10)$$

It can be verified easily by simple calculation that (9)

[4] Konishi analyzed a 2-port ferriresonator filter taking account of higher order modes [9].

[5] From (2) and (3) we get, $b_{11} = \Gamma - T = (1/\Delta')(Z_+Z_- - 1)$, $b_{21} = S_{21} - S_{12} = (1/\Delta')j(Z_- - Z_+)$, where $\Delta' = (1 + Z_+)(1 + Z_-)$.

and (10) in the case where $\alpha = 0$ are identical to the expressions given by Carter [5]. For the case where the RF losses in the ferrite resonator are negligible, he has discussed in detail the IL characteristics in the passband as a function of ω/ω_r (where $\omega_r = \omega_0$) with some parameters [5].

It is very important in particular at UHF frequencies, to study the characteristics for the case where the loss term α of the resonator cannot be neglected. In previous reports [6], [7], however, the relations between filter characteristics and physical properties of the resonator have not been explained sufficiently. In this paper we will investigate the characteristics of the circuit, particularly at the resonance frequency. Discussions from this point of view are of use and interest for designs and to find the limitations on filter circuits of this type.

2) *Detuning:* From the conditions of the minimum IL in the passband[6] $((d/d\omega)P_+ = 0)$ and $k_r^2(Q_eQ_f)^{-2} \ll 1$, the resonant frequency f_r can be determined by the following relation:

$$f_r = f_t\left(1 - \frac{k_r - \alpha^2 k_r^2}{Q_e^r Q_f}\right)^{-1} \simeq f_t\left(1 + \frac{k_t}{Q_e^t Q_f}\right) \quad (11)$$

where Q_e^i is the value of Q_e at $\omega = \omega_i (i = r,t)$. Therefore, the detuning from f_t goes through a maximum at a value of $k_t = Q_f$, after which it decreases with increasing k_t.

3) *Analysis at Resonance:* When a unit input is applied, the IL in the passband IL^r, the reflection at input port $|b_{11}^r|^2$, and the net dissipation loss of the resonator L^r at the center frequency are, respectively, expressed as follows:

$$\mathrm{IL}^r = (1 + \alpha Q_e)^2 \left[1 + \frac{(1 + \alpha Q_e)^2}{4Q_e^2}\right] \quad (12a)$$

$$|b_{11}^r|^2 = \frac{(1 - \alpha^2 Q_e^2)^2 + 4\alpha^2 Q_e^4}{(1 + \alpha Q_e)^2[4Q_e^2 + (1 + \alpha Q_e)^2]} \quad (12b)$$

$$L^r = \frac{4\alpha Q_e[2Q_e^2 + (1 + \alpha Q_e)^2]}{(1 + \alpha Q_e)^2[4Q_e^2 + (1 + \alpha Q_e)^2]}. \quad (12c)$$

a) $\alpha = 0$. In this case we get $\mathrm{IL}^r = 1 + 1/(4Q_e^2)$, $|b_{11}^r|^2 = (1 + 4Q_e^2)^{-1}$, and $L^r = 0$. Note that the center frequency IL in the passband goes through a maximum at a value of $R_0 = \omega_r L_c$, after which it decreases with increasing R_0:

$$\mathrm{IL}_{\max}^r = 1 + \frac{1}{16Q_f^2}. \quad (13)$$

b) $\alpha Q_e = 1$. In this case (12a)–(12c) show that a phenomemon of interest occurs: $|b_{11}^r|^2 = |b_{21}^r|^2 \simeq 1/4$ and L^r has a maximum: $L_{\max}^r \simeq 1/2$. Therefore, on designing filters of this type, caution should be taken that the external Q of the filter circuit $(Q_e/2)$ is not too close to an unloaded Q of the magnetic resonator ($Q_u = 1/2\alpha$). Note that this phenomenon of interest may be observed not only in a 2-port magnetic-resonator BPF but also in a 4-port magnetic-resonator circulator.

4) For the general case where a loss term α of the magnetic resonator is fixed as a parameter, the IL IL^r, the net dissipation loss L^r, and the inverse of the reflection at input port $|b_{11}^r|^{-2}$ are shown as a function of αQ_e in Fig. 7. As is obvious from Fig. 7, no results obtained from analysis for the special case where $\alpha = 0$ can show the properties of the IL^r, L^r, and $|b_{11}^r|^{-2}$ in higher regions of the external Q, Q_e. Consequently, it may be impossible to precisely understand limitations to characteristics of magnetic resonator circuits (we will discuss these in detail, in Section V).

In previous reports, the IL in the passband IL^r is independent of external Q and always equal to one in the case where $\alpha = 0$, or it approaches one with decreasing Q_e in the case where $\alpha \neq 0$ [6], [10].[7] However, this can hardly hold good in lower regions of external Q. That is, it may be understood physically that the Q_e of a magnetic resonator should have the optimum value to minimize the IL^r; just because the output $|b_{21}^r|^2$ approaches zero, while most of the input signal becomes reflected at the input port as $Q_e \to 0$. Fig. 7 shows this fact clearly.

When an unloaded Q, Q_u, of a magnetic resonator is given, there exists the optimum external Q of the resonator to minimize the center frequency IL IL^r as shown in Fig. 7. Such an external Q, Q_e, required for the adequate low IL^r, however, is obtained over a pretty wide region in the case of a resonator with small α. The characteristics of the net dissipation loss L^r obviously have a similar tendency in the case of a 4-port circuit. This tendency is understood to be quite general.

It is obvious from Fig. 7 that the characteristics of the filter depend strongly upon the relation between Q_e and α (or Q_u), and that a small value of α and an adequately small value of αQ_e are desired. Since the production αQ_e is in inverse proportion to the production of unloaded Q and saturation magnetization, it is, in practice, of great importance for a magnetic resonator to have a large value of $Q_u \cdot M_s$.

5) *Fractional 3-dB Bandwidth:* It is useful to show the general formula of the bandwidth and to investigate the bandwidth by varying the external load R_0. From (9), the fractional 3-dB bandwidth of the resonator BPF is given by

$$\frac{\Delta f}{f_r} = 2\frac{f_r}{f_t}\left(\frac{1}{Q_e} + \alpha\right) \quad (14)$$

where Δf is the bandwidth between 3-dB attenuation points. Equation (14) is used in the case of the 4-port circuit mentioned in Section III, if Q_e is replaced with Q_f.

As understood from (14) and the notations Q_e and x, the fractional 3-dB bandwidth $\Delta f/f_r$ increases quasi-linearly with increasing M_s, while the geometric structure

[6] From $(d/d\omega_t)|P_+| = 0$, we get $f_r = f_t(1 - k_r/Q_e^r Q_f)^{-1}$, where the IL is a function of ω_t for a given ω_r.

[7] For instance, according to [10], $\mathrm{IL}^r = (1 + \alpha Q_e)^2$, under conditions and with notations mentioned above.

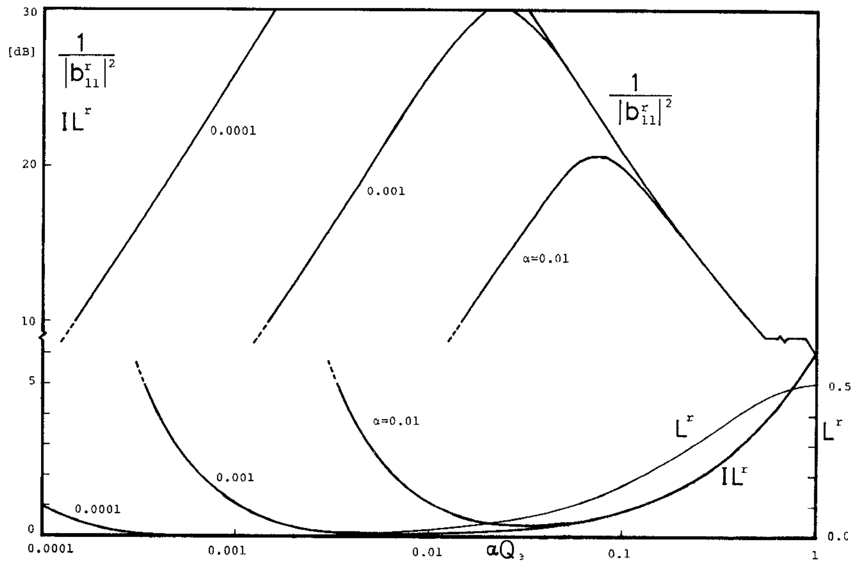


Fig. 7. IL, dissipation loss, and reflection of 2-port BPF as a function of αQ_e at resonance.

of the coupling circuit, the shape and volume of the resonator, and the external resistance R_0 are held constant (i.e., A is constant, R_0 is constant).[8] Note that the maximum IL in lower regions of external Q, Q_e, occurs simultaneously with the maximum fractional 3-dB bandwidth.

V. LIMITATIONS OF $\Delta f/f_r$ AND AVAILABLE RANGE

It is of great importance in theory and practice to clarify the limitations of available frequency range and 3-dB bandwidth of magnetic resonator circuits and also to discuss how a loss term α of a resonator or an external Q of a resonator should be restricted in order to satisfy the requirements of the center frequency IL (or RL) and the reflection (in the case of 2-port filter).

A. 4-Port Circulator

1) *Condition of α, k_r, and Q_f:* We assume that the center frequency IL in the passband (or the center frequency RL in the stopband) and that in the stopband (or the center frequency RL in the passband, or the center frequency reflection in the case of the 2-port filter) are, respectively, required to be as follows:

$$IL_{BPF}{}^r,\ RL_{BSF}{}^r \le n^2 \qquad IL_{BSF}{}^r,\ RL_{BPF}{}^r \ge N^2 \quad (15)$$

where $2 \ge n \ge 1$, $N^2 \gg 1$. In order that this 4-port nonreciprocal circuit operates as a circulator, it is necessary to satisfy the conditions: $|Z_+{}^r|^2 \ge (N/n)^2$ and $|Z_-{}^r|^2 \le (n/N)^2$, in another expression, $(\alpha Q_f)^{-1} \ge N/n$ and $k_r/Q_f \le n/N$. Also, in order to meet the requirement (15) of a 4-port circulator, from (7a) and (7b) and (8a) and (8b), the necessary condition for Q_f is given by

$$\max\left\{\frac{1}{(n^2-1)^{1/2}},\ (N^2-1)^{1/2}\right\}\cdot(k_r+\tfrac{1}{2}) \lesssim Q_f \lesssim \min\left\{\frac{n-1}{\alpha},\ \frac{1}{\alpha(N-1)}\right\}. \quad (16a)$$

Note the effect of k_r. In the case where $n \gtrsim 1 + 1/(N-1)$ and $k_r \gg 1/2$, inequality (16a) becomes the simpler form (16b):

$$Nk_r \lesssim Q_f \lesssim \frac{1}{\alpha N}. \quad (16b)$$

2) *Limitation of $\Delta f/f_r$:* Under the condition restricted by (15), the lower and upper limitation of the fractional 3-dB bandwidth are approximately given by the inequality, using (14) (where Q_e is replaced with Q_f) and (16a). In the case where (16b) can be used, the limitations of $\Delta f/f_r$ are given in a simple form by

$$\frac{N}{Q_u} \lesssim \frac{\Delta f}{f_r} \lesssim \frac{2}{Nk_r} + \frac{1}{Q_u}. \quad (17)$$

3) *Limitation of Available Frequency Range:* In the case of the ellipsoidally shaped resonator, it is a well known fact that the lower limitation f_L of the magnetically tunable range is given by $f_L > N_t f_m$, where $f_m = \omega_m/2\pi$. The unloaded Q, Q_u, of the resonator depends upon both its geometrical shape and frequency. That is, the unloaded Q for low resonant frequencies is written as follows [11]:

$$Q_u = (\omega_t - N_z\omega_m)\cdot\frac{\tau}{2} \quad (18)$$

where N_z is the demagnetizing factor in the z direction, τ is the Bloch–Bloembergen phenomenological relaxation time (also see [6]). Thus the unloaded Q, Q_u, becomes equal to zero at the lower limitation frequency f_L.

The coupling impedance x is considered to be inde-

[8] In experimental results by Young and Weller [8], the 3-dB bandwidths of the pure YIG filter were approximately twice that of the GaYIG filter, notwithstanding the same filter structure. This may be understood from a two-fold increase of saturation magnetization; $4\pi M_s$ of pure YIG = 1780 (Oe), $4\pi M_s$ of GaYIG = 1000 (Oe).

pendent of frequency over an extremely wide range. Accordingly, the coupling impedance x can be regarded as constant for frequencies. The directional coupler with coupled transmission lines has a very wide bandwidth if it is adequately designed [12]. Therefore, we get the upper limitation f_H of the available frequency range after consideration of (16a):

$$f_H \lesssim 2 \frac{A}{L_c} f_m Q_u^* \cdot \min \left\{ n-1, \frac{1}{N-1} \right\} \cdot \min \left\{ (n^2-1)^{1/2}, \frac{1}{(N^2-1)^{1/2}} \right\} \quad (19)$$

where Q_u^* is the unloaded Q of resonator at frequency f_H. The center frequency IL's and the center frequency RL's, in the case where k_r increases from 10 to 80, are shown in Fig. 8.

B. 2-Port BPF

1) *Condition of α and Q_e:* Taking into account that the ratio of output power to reflection power at the input port has a maximum value of $1/\alpha$ when $Q_e = Q_u^{1/2}$, the requirement (15) can be expressed by means of the following inequality:

$$\frac{1}{\alpha} \gtrsim \frac{|b_{21}^r|^2}{|b_{11}^r|^2} \geq \left(\frac{N}{n}\right)^2. \quad (20)$$

Therefore, after due consideration of (12a) and (12b) (for a given N, a minimum Q_e increases with increasing α), in order to meet the requirement (15) of the BPF, the condition necessary for the external Q, Q_e, is approximately given by the following inequality:

$$\max \left\{ \frac{1}{2(n^2-1)^{1/2} - \alpha}, \frac{N}{2-\alpha N} \right\} \lesssim Q_e \lesssim \min \left\{ \frac{n-1}{\alpha}, \frac{1}{\alpha(N-1)} \right\}. \quad (21)$$

2) *Limitation of $\Delta f/f_r$:* Under the condition restricted by (15), the fractional 3-dB bandwidth cannot go beyond the conclusions of (14) and (21). The limitations of the fractional 3-dB bandwidth are approximately given by

$$\max \left\{ \frac{2\alpha n}{n-1}, 2\alpha N \right\} \lesssim \frac{\Delta f}{f_r} \lesssim \min \left\{ 4(n^2-1)^{1/2}, \frac{4}{N} \right\}. \quad (22)$$

3) *Limitation of Available Frequency Range:* The lower limitation f_L for the 2-port resonator filter is given by $f_L > N_t f_m$ as discussed above. As is obvious from (4c), when Q_e is fixed, k^2 has a maximum value of $Q_e^2/4$ at $Q_f = Q_e/2$. Thus we get $k \leq Q_e/2$. Therefore, the upper limitation f_H of the available frequency range is given, after consideration to (21), by

$$f_H \lesssim \frac{A}{L_c} f_m Q_u^* \cdot \min \left\{ n-1, \frac{1}{N-1} \right\}. \quad (23)$$

The lower limitation f_L can be lowered by using a disk-shaped resonator and a low-M_s resonator [8]. As is obvious from (19) or (23), the upper limitation f_H can be extended by using a resonator which has a large value of $Q_u \cdot M_s$ and by designing a coupling structure which has a large ratio of A to L_c. Examples of calculations are shown in Fig. 9.

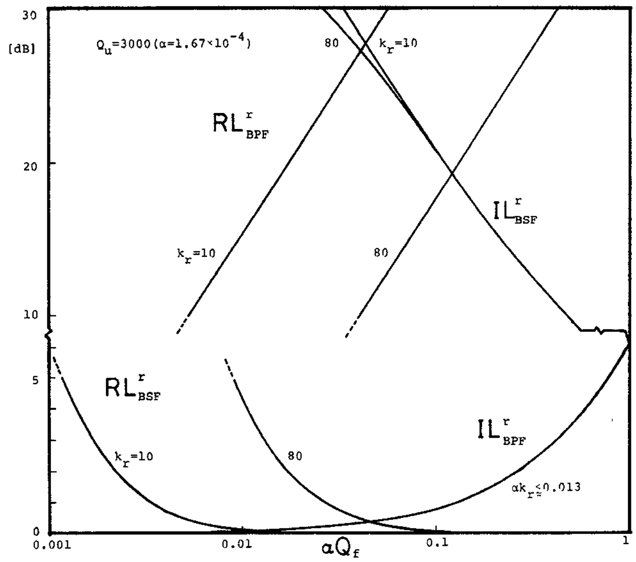


Fig. 8. IL's and RL's of the 4-port nonreciprocal circuit in the case where k_r increases from 10 to 80.

VI. CONCLUSION

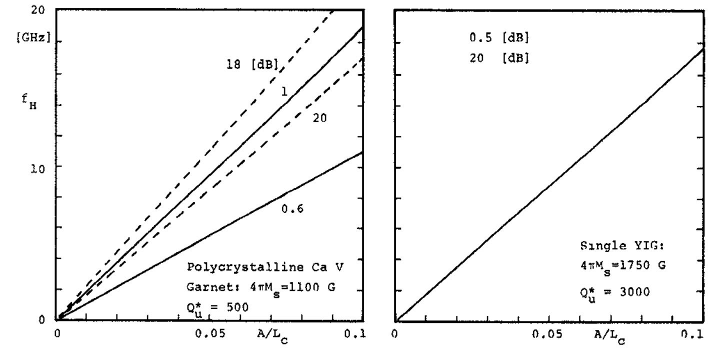
The general formulas for 4-port and 2-port nonreciprocal circuit responses are obtained. The maximum dissipation loss of the magnetic resonator on resonance is just one half of the input power. An optimum value of the external Q of the resonator exists. The requirements for the center frequency IL or RL and the center frequency reflection are introduced quantitatively and the conditions necessary for loss term α, k_r, and external Q are obtained. Moreover, the limitations of the fractional 3-dB bandwidth and the available frequency range are obtained. Thus the basic principle of design for the magnetic resonator circuit is made clear. The results obtained from the complete analysis show that even a polycrystalline magnetic resonator could be useful for the nonreciprocal circuit of this type, if the product of unloaded Q, Q_u, and saturation magnetization M_s has a sufficiently large value.

APPENDIX

$$A_{ij} = \frac{\mu_0 V_f C_j}{2\pi} \int_{S_i} \frac{1}{r^3} dS \text{ (H)}$$

where $i = x, y$, $j = x, y$, V_f is the volume of magnetic resonator, r is the distance from the center of the resonator, S_i is the integral region concerning i line, and C_j is the ratio of field produced at the center of the resonator to current flowing on the j line in the absence of the resonator. For example, from Fig. 10 we get

$$A_{xy} = \frac{\mu_0 V_f}{2\pi} I_x C_y$$



(a) (b)

Fig. 9. (a) f_H of 2-port BPF. (b) f_H of 4-port resonance-type circulator. Upper limitations of available frequency range. (a) f_H for $IL^r \leqq 0.6$ dB (or 1 dB) and for the inverse of the reflection $\geqq 18$ dB (or 20 dB). (b) f_H for $IL_{BPF}{}^r$, $RL_{BSF}{}^r \leqq 0.5$ dB and for $IL_{BSF}{}^r$, $RL_{BPF}{}^r \geqq 20$ dB.

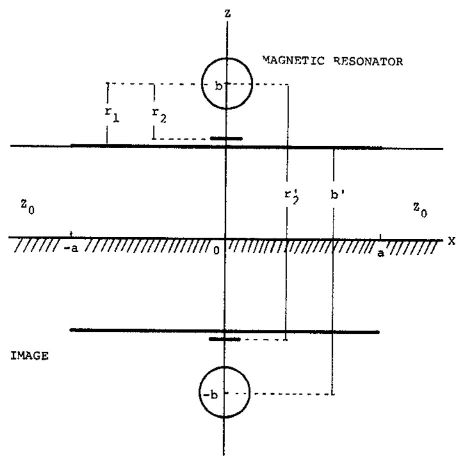


Fig. 10. Sectional view of X-form orthogonal coupling.

$$I_x = 2\left\{\left(\frac{1}{r_1{}^2} + \frac{1}{a^2}\right)^{1/2} - \left(\frac{1}{a^2} + \frac{1}{b'^2}\right)^{1/2}\right\}$$

$$C_y = \frac{1}{2\pi}\left(\frac{1}{r_2} - \frac{1}{r_2'}\right).$$

Refer to [4]–[7] in the case of the 2-port BPF in Fig. 3.

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